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Application of a New Family of Functions on the Space of Analytic Functions

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Abstract

In this paper we investigate a family of integral operators defined on the space of analytic functions. By making use of these novel integral operators we give some applications of the new families of analytic functions on the same space associated with quasi-Hadamard product in the unit disk $\mathbb{U}$.

key words. Analytic Functions, Differential Operator, Subordination.

AMS subject classifications. 30C45

1 Introduction and Preliminaries

Let H be the class of functions analytic in $\mathbb{U} = \{z : |z| < 1\}$ and $H[a, n]$ be the subclass of H consisting of functions of the form $f(z) = a+a_n z^n + a_{n+1}z^{n+1} + a_{n+2}z^{n+2} + \cdots$. Let $A_p \subseteq H[a, n]$ denote the class of all functions of the form $f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k$, $p \in \mathbb{N} = \{1, 2, \cdots\}$.

Let A denote the class of functions of the form $f(z) = z+a_2z^2+a_3z^3+\cdots$ or $f(z) = z+\sum_{n=2}^{\infty} a_n z^n$ which are analytic in the open unit disk $\mathbb{U} = \{z : |z| < 1\}$ normalized by $f(0) = f'(0) - 1 = 0$.

For functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$ the Hadamard product (or convolution) $f * g$ is defined by $(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n$.

For a function $f \in A_p$, we define a linear differential operator as follow:

$$\begin{aligned}
\Upsilon^0 f(z) &= f(z);\\
\Upsilon^1_\lambda(p, \alpha, \beta, \mu) f(z) &= \left(\frac{\alpha - p\mu + \beta - p\lambda}{(\alpha + \beta)}\right) f(z) + \left(\frac{p\mu + p\lambda}{(\alpha + \beta)}\right)\frac{zf'(z)}{p};\\
\Upsilon^2_\lambda(p, \alpha, \beta, \mu) f(z) &= D(\Upsilon^1_\lambda(p, \alpha, \beta, \mu) f(z));\\
&\vdots\\
\Upsilon^n_\lambda(p, \alpha, \beta, \mu) f(z) &= z^p + \sum_{k=p+1}^{\infty}\left(\frac{\alpha + (\mu + \lambda)(k - p) + \beta}{\alpha + \beta}\right)^n a_k z^k. \qquad (1)
\end{aligned}$$

$(\alpha, \beta, \mu, \lambda \geq 0, \alpha + \beta \neq 0, n \in \mathbb{N}_o = \{0, 1, \cdots\})$. The operator $\Upsilon^n_\lambda(p, \alpha, \beta, \mu)$ is the generalized form of the following operators introduced by well know authors such as:

1. $p = 1,\ \Upsilon^n_\lambda(\alpha, \beta, \mu)f = z + \sum_{k=2}^{\infty}(\frac{\alpha+(\mu+\lambda)(k-1)+\beta}{\alpha+\beta})^n a_k z^k$(see [8],[9],[10]);
2. $p = 1\, \beta = 0\ \Upsilon^n_\lambda(1, \alpha, 0, \mu)f = \Upsilon^n_\lambda(\alpha, \mu)f = z + \sum_{k=2}^{\infty}(\frac{\alpha+(\mu+\lambda)(k-1)}{\alpha})^n a_k z^k$(see[7]);
3. $\alpha = p,\ \mu = 0\ \Upsilon^n_\lambda(p, \ell, 0)f = I^n_p(\lambda, \ell)f = z^p + \sum_{k=2}^{\infty}(\frac{p+\lambda(k-p)+\ell}{p+\ell})^n a_k z^k$(see[4]);
4. $\alpha = p,\ \mu = 0,\ \lambda = 1\ \Upsilon^n_\lambda(p, \ell) = z^p + \sum_{k=2}^{\infty}(\frac{k+\ell}{p+\ell})^n a_k z^k$(see[13],[18]);
5. $, p = \beta = 1,\ \mu = 0\ \Upsilon^n_\lambda(1, \alpha, 1, 0)f = D^n f = z + \sum_{k=2}^{\infty}(\frac{\alpha+\lambda(k-1)+1}{\alpha+1})^n a_k z^k$(see[2]);
6. $p = \alpha = 1,\ \beta = o\ \Upsilon^n_\lambda f = D^n f = z + \sum_{k=2}^{\infty}(1 + \lambda(k-1))^n a_k z^k$(see[1]);
7. $p = \alpha = \lambda = 1,\ \beta = \mu = 0\ \Upsilon^n_1(1, 1, 0, 0)f = D^n f = z + \sum_{k=2}^{\infty}(k)^n a_k z^k$(see[16]);
8. $p = \alpha = \beta = \lambda = 1,\ \mu = 0\ \Upsilon^n_1(1, 1, 1, 0)f(z) = D^n f = z + \sum_{k=2}^{\infty}(\frac{k+1}{2})^n a_k z^k$(see[19]);
9. $p = \beta = \lambda = 1, \mu = 0,\ \Upsilon^n_1(\alpha)f = z + \sum_{k=2}^{\infty}(\frac{k+\alpha}{\alpha+1})^n a_k z^k$(see[5],[6]).

Similarly for $f \in A_p$ on the space of multivalent analytic functions we define an integral operator as follows:

$$
\begin{aligned}
\mathtt{C}^0_p(\alpha, \beta, \mu, \lambda)f(z) &= f(z);\\
\mathtt{C}^1_p(\alpha, \beta, \mu, \lambda)f(z) &= \frac{\alpha+\beta}{\mu+\lambda} z^{p-(\frac{\alpha+\beta}{\mu+\lambda})} \int_o^z t^{(\frac{\alpha+\beta}{\mu+\lambda})-p-1} f(t)dt;\\
\mathtt{C}^2_p(\alpha, \beta, \mu, \lambda)f(z) &= \frac{\alpha+\beta}{\mu+\lambda} z^{p-(\frac{\alpha+\beta}{\mu+\lambda})} \int_o^z t^{(\frac{\alpha+\beta}{\mu+\lambda})-p-1} \mathtt{C}^1_p(\alpha, \beta, \mu, \lambda)f(t)dt;\\
&\vdots\\
\mathtt{C}^m_p(\alpha, \beta, \mu, \lambda)f(z) &= z^p + \sum_{k=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(k-p)+\beta})^m a_k z^k,\ m \in \mathbb{N}_0.
\end{aligned}
$$

The operator $\mathtt{C}^m_p(\alpha, \beta, \mu, \lambda)$ is also generalized form of the following operators:

1. $\mathtt{C}^\alpha_1(1, 1, 0, 1)f(z) = I^\alpha f(z) = z + \sum_{k=2}^{\infty}(\frac{2}{k+1})^\alpha a_k z^k$(see[12]);
2. $\mathtt{C}^m_p(p, 1, 0, 1)f(z) = I^m_p f(z) = z^p + \sum_{k=p+1}^{\infty}(\frac{p+1}{k+1})^m a_k z^k$(see[17]);
3. $\mathtt{C}^m_1(1, 0, 0, \lambda)f(z) = I^{-m}_\lambda f(z) = z + \sum_{k=2}^{\infty}(1 + \lambda)(k-1))^{-m} a_k z^k$(see[14]);
4. $\mathtt{C}^\alpha_1(1, 1, 0, 1)f(z) = I^\alpha f(z) = z + \sum_{k=2}^{\infty}(\frac{2}{k+1})^\alpha a_k z^k$(see[11]);

5. $\mathtt{C}_p^m(p,1,0,1)f(z) = I_p^m f(z) = z^p + \sum_{k=p+1}^{\infty}(\frac{p+1}{k+1})^m a_k z^k$(see[15]);

6. $\mathtt{C}_1^m(1,0,0,1)f(z) = I^m f(z) = z + \sum_{k=2}^{\infty}(k)^{-m} a_k z^k$(see[16]);

7. $\mathtt{C}_p^m(p,\ell,0,\lambda)f(z) = J_p^m(\lambda,\ell)f(z) = z^p + \sum_{k=p+1}^{\infty}(\frac{p+\ell}{p+\ell+\lambda(k-p)})^m a_k z^k$(see[3]).

By specializing the parameters of $\mathtt{C}_p^m(\alpha,\beta,\mu,\lambda)$, we introduce certain new differential operators on the space of multivalent analytic functions as follow:

1. $\mathtt{C}_p^m(\alpha,0,\mu,\lambda)f(z) = \mathtt{C}_p^m(\alpha,\mu,\lambda)f(z) = z^p + \sum_{k=2}^{\infty}(\frac{\alpha}{\alpha+(\mu+\lambda)(k-p)})^m a_k z^k$;
2. $\mathtt{C}_p^m(p,0,\mu,\lambda)f(z) = \mathtt{C}_p^m(\mu,\lambda)f(z) = z^p + \sum_{k=2}^{\infty}(\frac{p}{p+(\mu+\lambda)(k-p)})^m a_k z^k$;
3. $\mathtt{C}_p^m(\alpha,0,0,\lambda)f(z) = \mathtt{C}_p^m(\alpha,\lambda)f(z) = z^p + \sum_{k=2}^{\infty}(\frac{\alpha}{\alpha+\lambda(k-p)})^m a_k z^k$;
4. $\mathtt{C}_p^m(\alpha,\beta,0,\lambda)f(z) = \mathtt{C}_p^m(\alpha,\beta,\lambda)f(z) = z^p + \sum_{k=2}^{\infty}(\frac{\alpha+\beta}{\alpha+\lambda(k-p)+\beta})^m a_k z^k$;
5. $\mathtt{C}_p^m(\alpha,\beta,0,1)f(z) = \mathtt{C}_p^m(\alpha,\beta)f(z) = z^p + \sum_{k=2}^{\infty}(\frac{\alpha+\beta}{\alpha+k-p+\beta})^m a_k z^k$;
6. $\mathtt{C}_p^m(\alpha,0,0,1)f(z) = \mathtt{C}_p^m(\alpha)f(z) = z^p + \sum_{k=2}^{\infty}(\frac{\alpha}{\alpha+k-p})^m a_k z^k$;
7. $\mathtt{C}_p^m(1,0,0,1)f(z) = \mathtt{C}_p^m f(z) = z^p + \sum_{k=2}^{\infty}(\frac{1}{1+k-p})^m a_k z^k$;
8. $\mathtt{C}_p^m(1,1,0,\lambda)f(z) = \mathtt{C}_p^m(\lambda)f(z) = z^p + \sum_{k=2}^{\infty}(\frac{2}{2+\lambda(k-p)})^m a_k z^k$;
9. $\mathtt{C}_p^m(p,\ell,\mu,\lambda)f(z) = \mathtt{C}_p^m(\ell,\mu,\lambda)f(z) = z^p + \sum_{k=2}^{\infty}(\frac{p+\ell}{p+(\mu+\lambda)(k-p)+\ell})^m a_k z^k$;
10. $\mathtt{C}_p^m(1,\beta,0,1)f(z) = \mathtt{C}_p^m(\beta)f(z) = z^p + \sum_{k=2}^{\infty}(\frac{p+\beta}{k+\beta})^m a_k z^k$;
11. $\mathtt{C}_p^m(0,\ell,\mu,\lambda)f(z) = \mathtt{C}_p^m(\ell,\mu,\lambda)f(z) = z^p + \sum_{k=2}^{\infty}(\frac{\ell}{(\mu+\lambda)(k-p)+\ell})^m a_k z^k$.

Throughout this paper, we consider the functions of the form as follow

$$f(z) = a_1 z + \Sigma_2^{\infty} a_n z^n, \ (a_1 > 0, a_n \geq 0), \tag{2}$$
$$f_i(z) = a_{1,i} z + \Sigma_2^{\infty} a_{n,i} z^n, \ (a_{1,i} > 0, a_{n,i} \geq 0), \tag{3}$$
$$g(z) = b_1 z + \Sigma_2^{\infty} b_n z^n, \ (b_1 > 0, b_n \geq 0), \tag{4}$$
$$g_j(z) = b_{1,j} z + \Sigma_2^{\infty} b_{n,j} z^n, \ (b_{1,i} > 0, b_{n,j} \geq 0), \tag{5}$$

be regular and univalent in the unit disc $\mathbb{U} = \{z : |z| < 1\}$·

For Convenience, we take $\mathtt{C}_1^m(\alpha,\beta,\mu,\lambda)f(z) = \mathtt{C}^m$

For $0 \leq \rho < 1$, $0 \leq \delta < 1$ and $\eta \geq 0$, we let $\mathfrak{J}(k, \rho, \delta, \eta)$ denote the class of functions f defined by (2) and satisfying the analytic criterion

$$\Re\{\frac{z(\mathtt{C}^m)'}{(1-\rho)(\mathtt{C}^m) + \rho z(\mathtt{C}^m)'} - \delta\} \quad > \quad \eta\{\frac{z(\mathtt{C}^m)'}{(1-\rho)(\mathtt{C}^m) + \rho z(\mathtt{C}^m)'} - 1\}.$$

Also let $\mathfrak{C}(k, \rho, \delta, \eta)$ denote the class of functions f defined by (2) and satisfying the analytic criterion

$$\Re\{\frac{(\mathtt{C}^m)' + z(\mathtt{C}^m)''}{(\mathtt{C}^m)' + \rho z(\mathtt{C}^m)''} - \delta\} \quad > \quad \eta\{\frac{(\mathtt{C}^m)' + z(\mathtt{C}^m)''}{(\mathtt{C}^m)' + \rho z(\mathtt{C}^m)''} - 1\}.$$

A function $f \in \mathfrak{J}(k, \rho, \delta, \eta)$ $(0 \leq \rho < 1, 0 \leq \delta < 1, \eta \geq 0)$ if and only if

$$\sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^k[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]|a_{n,i}| \quad \leq \quad (1-\delta)|a_{1,i}|,$$

and $f \in \mathfrak{C}(n, \rho, \delta, \eta)$ $(0 \leq \rho < 1, 0 \leq \delta < 1, \eta \geq 0)$ if and only if

$$\sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^{k+1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]|a_{n,i}| \quad \leq \quad (1-\delta)|a_{1,i}|.$$

A function f which is analytic in U belonging to the class $\mathfrak{R}_p(k, \rho, \delta, \eta)$ if and only if

$$\sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^{p}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]|a_{n,i}| \leq (1-\delta)|a_{1,i}|, \tag{6}$$

where $0 \leq \rho < 1$, $0 \leq \delta < 1$, $\eta \geq 0$ and p is any fixed nonnegative real number.

For $p = k$ and $p = k + 1$, it is identical to the family of functions denoted by $\mathfrak{J}(k, \rho, \delta, \eta)$ and $\mathfrak{C}(k, \rho, \delta, \eta)$ respectively. Further, for any positive integer $p > h > h - 1 > \cdots > k + 1 > k$, we have the inclusion relation

$$\mathfrak{J}(k, \rho, \delta, \eta) \subseteq \mathfrak{C}(k, \rho, \delta, \eta) \subseteq \cdots \subseteq \mathfrak{R}_h(k, \rho, \delta, \eta) \subseteq \mathfrak{R}_p(k, \rho, \delta, \eta).$$

The class $\mathfrak{R}_p(n, \rho, \delta, \eta)$ is nonempty for any nonnegative real number p as the functions of the form $f(z) = a_1 z + \sum_{n=2}^{\infty} \frac{(\alpha+(\mu+\lambda)(n-1)+\beta)^p(1-\delta)}{(\alpha+\beta)^p[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]}\lambda_n z^n$, where $a_1 > 0$, $\lambda_n \geq 0$ and $\Sigma_2^{\infty}\lambda_n \leq 1$; satisfy the inequality (6).

2 Main Results

Theorem 2.1. Let the functions f_i defined by (3) belonging to the family of functions $\mathfrak{C}(k, \rho, \delta, \eta)$ defined on space of analytic functions for all by $i = 1, 2, \cdots r$. Then quasi-Hadamard product of $f_1 * f_2 * \cdots * f_r$ belongs to the family $\mathfrak{R}_{r(k+2)-1}(n, \rho, \delta, \eta)$ on same space of analytic functions.

Proof Since $f_i \in \mathfrak{C}(k,\rho,\delta,\eta)$, implies

$$\sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^{k+1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]|a_{n,i}| \leq (1-\delta)|a_{1,i}|, \qquad (7)$$

$\Rightarrow$

$$|a_{n,i}| \leq (\frac{(\alpha+\beta)}{(\alpha+(\mu+\lambda)(n-1)+\beta)})^{-k-2}|a_{1,i}|, \forall\, i=1,2,\cdots r. \qquad (8)$$

Using (7) as well as (8) for $i=r$ and $i=1,2,\cdots r-1$, implies

$$\sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^{r(k+2)-1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\prod_{i=1}^{r}|a_{n,i}| \leq$$
$$\sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^{k+1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]|a_{n,r}|\prod_{i=1}^{r-1}|a_{1,i}| =$$
$$(1-\delta)\prod_{i=1}^{r}|a_{1,i}| \Rightarrow f_1 * f_2 * \cdots * f_r \in \mathfrak{R}_{r(k+2)-1}(k,\rho,\delta,\eta) \quad .$$

Theorem 2.2. Let the functions f_i defined by (3) belonging to the family $\mathfrak{J}(k,\rho,\delta,\eta)$ of functions on space of analytic functions for all by $i=1,2,\cdots r$. Then quasi-Hadamard product product of $f_1 * f_2 * \cdots * f_r$ belongs to the family $\mathfrak{R}_{r(k+1)-1}(n,\rho,\delta,\eta)$ on the same space of analytic functions.

Proof Using the same techniques of the proof of Theorem 2.1, we proved that

$$\sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^{r(k+1)-1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]\prod_{i=1}^{r}|a_{n,i}| \leq$$
$$\sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^{k+1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]|a_{n,r}|\prod_{i=1}^{r-1}|a_{1,i}| =$$
$$(1-\delta)\prod_{i=1}^{r}|a_{1,i}| \Rightarrow f_1 * f_2 * \cdots * f_r \in \mathfrak{R}_{r(k+1)-1}(k,\rho,\delta,\eta) \quad .$$

Theorem 2.3. Let the functions f_i defined by (3) belonging to a family $\mathfrak{C}(k,\rho,\delta,\eta)$ of functions on space of analytic functions for all by $i=1,2,\cdots r$ and let g_j defined by (5) belonging to family $\mathfrak{J}(n,\rho,\delta,\eta)$ of functions on space of analytic functions for all by $j=1,2,\cdots q$. Then quasi-Hadamard product of $f_1 * f_2 * \cdots * f_r * g_1 * g_2 * \cdots * g_q$ belongs to the class $\mathfrak{R}_{r(k+2)+q(k+1)-1}(n,\rho,\delta,\eta)$ on the same space of analytic functions.

Proof Let us denote $f_1 * f_2 * \cdots * f_r * g_1 * g_2 * \cdots * g_q$ by G. Then

$$G(z) = [\prod_{i=1}^{r}|a_{1,i}|][\prod_{j=1}^{q}|b_{1,j}|]z + \sum_{n=2}^{\infty}[\prod_{i=1}^{r}|a_{n,i}|][\prod_{j=1}^{q}|b_{n,j}|]z^n.$$

Since $f_i \in \mathfrak{C}(k,\rho,\delta,\eta)$ and $g_j \in \mathfrak{J}(k,\rho,\delta,\eta)$, implies

$$\sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^{k+1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]|a_{n,i}| \leq (1-\delta)|a_{1,i}|, \forall\, i=1,2,\cdots r.$$

$$\begin{aligned} |a_{n,i}| &\leq \frac{(\alpha+(\mu+\lambda)(n-1)+\beta))^{k+1}(1-\delta)|a_{1,i}|}{((\alpha+\beta))^{k+1}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]}, \\ |a_{n,i}| &\leq (\frac{(\alpha+\beta)}{(\alpha+(\mu+\lambda)(n-1)+\beta)})^{-k-2}|a_{1,i}|, \forall\, i=1,2,\cdots r. \qquad (9) \\ |b_{n,i}| &\leq (\frac{(\alpha+\beta)}{(\alpha+(\mu+\lambda)(n-1)+\beta)})^{-k-1}|a_{b,i}|, \forall\, i=1,2,\cdots q. \qquad (10) \end{aligned}$$

Also

$$\sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^{k}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]|b_{n,j}| \leq (1-\delta)|b_{1,j}|. \qquad (11)$$

Using (9), (11) and (10) for $i=1,2,\cdots,r$, $j=q$ and $j=1,2,\cdots q-1$ respectively. we have (consider $t=r(k+2)+q(k+1)-1$)

$$\begin{aligned} \sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^{t}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)][\prod_{i=1}^{r}|a_{n,i}|][\prod_{j=1}^{q}|b_{n,j}|] &\leq \\ \sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^{k}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)][|b_{n,q}|\prod_{i=1}^{r}|a_{1,i}|][\prod_{j=1}^{q-1}|b_{1,j}|] &= \\ \sum_{n=2}^{\infty}(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^{k}[n(1+\eta)-(\delta+\eta)(1+n\rho-\rho)]|b_{n,q}|[\prod_{i=1}^{r}|a_{1,i}|][\prod_{j=1}^{q-1}|b_{1,j}|] &\leq \\ (1-\delta)[\prod_{i=1}^{r}|a_{1,i}|][\prod_{j=1}^{q}|b_{1,j}|] &\Rightarrow \end{aligned}$$

$f_1 * f_2 * \cdots * f_r * g_1 * g_2 * \cdots * g_q \in \mathfrak{R}_{r(k+2)+q(k+1)-1}(n,\rho,\delta,\eta)$ as required.

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