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New inextensible flows of tangent developable surfaces in Euclidian 3-space $\mathbb{E}^3$

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Abstract

In this paper, we study inextensible flows of tangent developable surfaces in Euclidean 3-space $\mathbb{E}^3$. We obtain results for minimal tangent developable surfaces in Euclidean 3-space $\mathbb{E}^3$.

key words. Developable surface, Euclidean 3-space, inextensible flows.

AMS subject classifications. 53A04, 53A05.

1 Introduction

Developable surfaces are commonly used when manufacturing with materials that do not stretch or tear. Any process manipulating fabric, paper, leather, sheet metal or plywood will benefit from developable surface modeling techniques since these materials admit little distortion. In typical setups, the patterns of the product are first designed by trained individuals, with a computer performing a bending simulation to help forecast the manufactured result. The product is then fabricated by cutting out the patterns of the surface from a flat sheet of the respective material and bending these planar patterns to form the desired shape. Applications include modeling ship hulls, buildings, airplane wings, garments, ducts, automobile parts.

In this paper, we study inextensible flows of tangent developable surfaces in Euclidean 3-space $\mathbb{E}^3$. We obtain results for minimal tangent developable surfaces in Euclidean 3-space $\mathbb{E}^3$.

2 Preliminaries

The Euclidean 3-space $\mathbb{E}^3$ provided with the standard flat metric given by

$$\langle , \rangle = dx_1^2 + dx_2^2 + dx_3^2,$$

where (x_1, x_2, x_3) is a rectangular coordinate system of $\mathbb{E}^3$. Recall that, the norm of an arbitrary vector $a \in \mathbb{E}^3$ is given by $\|a\| = \sqrt{\langle a, a \rangle}$. γ is called a unit speed curve if velocity vector v of γ satisfies $\|a\| = 1$.

Denote by $\{\mathbf{T}, \mathbf{N}, \mathbf{B}\}$ the moving Frenet–Serret frame along the curve γ in the space $\mathbb{E}^3$. For an arbitrary curve γ with first and second curvature, κ and τ in the space $\mathbb{E}^3$, the following Frenet–Serret formulae is given

$$\begin{aligned} \mathbf{T}' &= \kappa \mathbf{N} \\ \mathbf{N}' &= -\kappa \mathbf{T} + \tau \mathbf{B} \\ \mathbf{B}' &= -\tau \mathbf{N}, \end{aligned}$$

where

$$\begin{aligned} \langle \mathbf{T}, \mathbf{T} \rangle &= \langle \mathbf{N}, \mathbf{N} \rangle = \langle \mathbf{B}, \mathbf{B} \rangle = 1, \\ \langle \mathbf{T}, \mathbf{N} \rangle &= \langle \mathbf{T}, \mathbf{B} \rangle = \langle \mathbf{N}, \mathbf{B} \rangle = 0. \end{aligned}$$

Here, curvature functions are defined by $\kappa = \kappa(s) = \|\mathbf{T}(s)\|$ and $\tau(s) = -\langle \mathbf{N}, \mathbf{B}' \rangle$.

Torsion of the curve γ is given by the aid of the mixed product

$$\tau = \frac{[\gamma', \gamma'', \gamma''']}{\kappa^2}.$$

3 Inextensible Flows of Tangent Developable Surfaces in the $\mathbb{E}^3$

A smooth surface X is called a developable surface if its Gaussian curvature K vanishes everywhere on the surface.

Definition 3.1. *(see [8])* A surface evolution $X(s, u, t)$ and its flow $\dfrac{\partial X}{\partial t}$ are said to be inextensible if its first fundamental form $\{E, F, G\}$ satisfies

$$\frac{\partial E}{\partial t} = \frac{\partial F}{\partial t} = \frac{\partial G}{\partial t} = 0.$$

This definition states that the surface $X(s,u,t)$ is, for all time t, the isometric image of the original surface $X(s,u,t_0)$ defined at some initial time t_0. For a developable surface, $X(s,u,t)$ can be physically pictured as the parametrization of a waving flag. For a given surface that is rigid, there exists no nontrivial inextensible evolution.

Definition 3.2. *We can define the following one-parameter family of tangent developable ruled surface*

$$X\left(s,u,t\right)=\gamma\left(s,t\right)+u\gamma'\left(s,t\right).$$

Theorem 3.3. *Let X is the tangent developable surface in $\mathbb{E}^3$. If $\frac{\partial X}{\partial t}$ is inextensible, then*

$$\frac{\partial \kappa}{\partial t}=0.$$

Proof. Assume that $X\left(s,u,t\right)$ be a one-parameter family of tangent developable surface. We show that X is inextensible.

$$\begin{aligned} X_s\left(s,u,t\right) &= \mathbf{T}+u\kappa\mathbf{N}, \\ X_u\left(s,u,t\right) &= \gamma'\left(s,t\right)=\mathbf{T}. \end{aligned}$$

If we compute first fundamental form $\{E,F,G\}$, we have

$$\begin{aligned} E &= \langle X_s, X_s\rangle = 1+u^2\kappa^2, \\ F &= \langle X_s, X_u\rangle = 1, \\ G &= \langle X_u, X_u\rangle = 1. \end{aligned}$$

Using above system, we have

$$\begin{aligned} \frac{\partial E}{\partial t} &= 2u^2\kappa\frac{\partial \kappa}{\partial t}. \\ \frac{\partial F}{\partial t} &= 0, \\ \frac{\partial G}{\partial t} &= 0. \end{aligned}$$

Also, $\frac{\partial X}{\partial t}$ is inextensible iff

$$\frac{\partial \kappa}{\partial t}=0.$$

Corollary 3.4. *Let X is the tangent developable surface in E^3. If flow of this developable surface is inextensible then this surface is minimal iff*

$$\tau = 0.$$

Proof. Using X_s and X_u, we get

$$\begin{aligned} X_{ss} &= -u\kappa^2\mathbf{T} + \left(\kappa + u\frac{\partial \kappa}{\partial s}\right)\mathbf{N} + u\kappa\tau\mathbf{B}, \\ X_{su} &= \kappa\mathbf{N}, \\ X_{uu} &= 0. \end{aligned}$$

Components of second fundamental form of developable surface are

$$\begin{aligned} h_{11} &= -u\kappa\tau, \\ h_{12} &= 0, \\ h_{22} &= 0. \end{aligned}$$

So, the mean curvature of one-parameter family of tangent developable surface $X(s,u,t)$ is

$$H = -\frac{\tau}{u\kappa}.$$

From above equation, X is a minimal surface in $\mathbb{E}^3$ if and only if

$$\tau = 0.$$

By the use of above equation the proof is complete.

Finally, we obtain lines of curvature.

If we put $\tan\theta = du/ds$, then

$$k_n(\theta) = \frac{h_{11}\cos^2\theta + 2h_{12}\cos\theta\sin\theta + h_{22}\sin^2\theta}{E\cos^2\theta + 2F\cos\theta\sin\theta + G\sin^2\theta}.$$

Using above calculations we get

$$k_n(\theta) = -\frac{u\kappa\cos^2\theta}{u^2\kappa^2\cos^2\theta + 2\cos\theta\sin\theta + 1}.$$

From definition of lines of curvature, we obtain

$$u\kappa \cos^2 \theta = 0.$$

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