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A Study of Certain New Subclasses defined in the Space of Analytic Functions

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Abstract

An attempt has been made to introduce certain new subclasses of analytic function bounds by some differential operator also define in the space of analytic functions. We study and investigate various inclusion properties of these classes. Some interesting applications of integral operators are also considered.

key words. Analytic functions, Integral operator, Inclusion properties.

AMS subject classifications. 30C45

1 Introduction and Preliminaries

Let A denote the class of analytic functions of the form $f(z) = z + \sum\limits_{n=2}^{\infty} a_n z^n$, in the open unit disk $\mathbb{U} = \{z : |z| < 1\}$ normalized by $f(0) = f'(0) - 1 = 0$.

For $f \in A$ and $\alpha, \beta, \mu, \lambda \geq 0$, author (cf.,[1, 2, 13]) introduced the following differential operator:

- $D_\lambda^n(\alpha, \beta, \mu) f(z) = z + \sum_{n=2}^{\infty} (\frac{\alpha+(\mu+\lambda)(n-1)+\beta}{\alpha+\beta})^n a_n z^n$.

Operator $D_\lambda^n(\alpha, \beta, \mu) f(z)$ is the generalized form of the following operators:

1. $D_\lambda^n(\alpha, 0, \mu) f(z) = D^n f(z)$ (see [3]);
2. $D_\lambda^n(\alpha, 1, 0) f(z) = D^n f(z)$ (see [4]);
3. $D_\lambda^n(1, 0, 0) f(z) = D^n f(z)$ (see [5]);
4. $D_1^n(1, 0, 0) f(z) = D^n f(z)$ (see [6]);
5. $D_1^n(1, 1, 0) f(z) = D^n f(z)$ (see [7]);

6. $D_1^n(\alpha,1,0)f(z) = D^n f(z)$ (see [8]).

Moreover we define a new integral operator as follows:

$$\begin{aligned}
\mathtt{C}^0(\alpha,\beta,\mu,\lambda)f(z) &= f(z);\\
\mathtt{C}^1(\alpha,\beta,\mu,\lambda)f(z) &= \frac{\alpha+\beta}{\mu+\lambda} z^{1-(\frac{\alpha+\beta}{\mu+\lambda})}\int_o^z t^{(\frac{\alpha+\beta}{\mu+\lambda})-1-1} f(t)dt;\\
\mathtt{C}^2(\alpha,\beta,\mu,\lambda)f(z) &= \frac{\alpha+\beta}{\mu+\lambda} z^{1-(\frac{\alpha+\beta}{\mu+\lambda})}\int_o^z t^{(\frac{\alpha+\beta}{\mu+\lambda})-1-1} \mathtt{C}^1(\alpha,\beta,\mu,\lambda)f(t)dt;\\
&\vdots\\
\mathtt{C}^m(\alpha,\beta,\mu,\lambda)f(z) &= z+\sum_{n=2}^{\infty}\left(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\right)^m a_n z^n. \qquad (1)
\end{aligned}$$

$\mathtt{C}^m(\alpha,\beta,\mu,\lambda)$ generalized some well know differential operators such that

1. $\mathtt{C}_1^\alpha(1,1,0,1)f(z) = I^\alpha f(z) = z+\sum_{n=2}^\infty \left(\frac{2}{n+1}\right)^\alpha a_n z^n$ (see [9]);

2. $\mathtt{C}_1^m(1,0,0,\lambda)f(z) = I_\lambda^{-m} f(z) = z+\sum_{n=2}^\infty \big(1+\lambda(n-1)\big)^{-m} a_n z^n$(see [10]);

3. $\mathtt{C}_1^\alpha(1,1,0,1)f(z) = I^\alpha f(z) = z+\sum_{n=2}^\infty \left(\frac{2}{n+1}\right)^\alpha a_n z^n$ (see [11]);

4. $\mathtt{C}_1^m(1,0,0,1)f(z) = I^m f(z) = z+\sum_{n=2}^\infty \big(n\big)^{-m} a_n z^n$, (see [12]).

Definition 1.1 Let $f\in A$ is said to belong to the class $M(\alpha,\beta,\mu,\lambda,\delta)$ if it satisfy the following analytic criterion

$$\Re\left(\frac{z(\mathtt{C}^m(\alpha,\beta,\mu,\lambda)f(z))'}{\mathtt{C}^m(\alpha,\beta,\mu,\lambda)f(z)}\right) > \delta, \quad 0\le\delta<1.$$

Definition 1.2 Let $f\in A$ is said to belong to the class $N(\alpha,\beta,\mu,\lambda,\delta)$ if it satisfy the following analytic criterion

$$\Re\left(\frac{(z(\mathtt{C}^m(\alpha,\beta,\mu,\lambda)f(z))')'}{(\mathtt{C}^m(\alpha,\beta,\mu,\lambda)f(z))'}\right) > \delta, \quad 0\le\delta<1.$$

2 Characterization properties

Theorem 2.1 If an analytic function $f\in A$ satisfy the following inequality

$$\sum_{n=2}^{\infty}(n-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m |a_n| \le 1-\delta, \quad 0\le\delta<1, \qquad (2)$$

then $f\in M(\alpha,\beta,\mu,\lambda,\delta)$.

Theorem 2.2 If an analytic function $f \in A$ satisfy the following inequality

$$\sum_{n=2}^{\infty} n(n-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m |a_n| \leq 1-\delta, \quad 0 \leq \delta < 1, \tag{3}$$

then $f \in N(\alpha,\beta,\mu,\lambda,\delta)$.

Proof Let $f \in M(\alpha,\beta,\mu,\lambda,\delta)$ implies

$$\Re\Big(\frac{z(\mathtt{C}^m(\alpha,\beta,\mu,\lambda)f(z))'}{\mathtt{C}^m(\alpha,\beta,\mu,\lambda)f(z)}\Big) > \delta, \quad \text{(see Definition 1.1)}.$$

Since we know that $\Re(w) > \alpha \Leftrightarrow |1-\alpha+w| > |1+\alpha-w|$. Therefore

$$\Big|(1-\delta)(\mathtt{C}^m(\alpha,\beta,\mu,\lambda)f(z)) + z(\mathtt{C}^m(\alpha,\beta,\mu,\lambda)f(z))'\Big| > \Big|(1+\delta)(\mathtt{C}^m(\alpha,\beta,\mu,\lambda)f(z)) - z(\mathtt{C}^m(\alpha,\beta,\mu,\lambda)f(z))'\Big|,$$

or

$$\Big|(2-\delta)z + \sum_{n=2}^{\infty}(1+n-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m a_n z^n\Big| - \Big|\delta z + \sum_{n=2}^{\infty}(1-n+\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m a_n z^n\Big| > 0.$$

Hence after doing some mathematics we have

$$\sum_{n=2}^{\infty}(2n-2\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m |a_n| \leq (2-2\delta),$$

or

$$\sum_{n=2}^{\infty}(n-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m |a_n| \leq (1-\delta).$$

This result is sharp with extremal function f given by

$$f(z) = z + \sum_{n=2}^{\infty} \frac{(1-\delta)}{(n-\delta)\big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\big)^m} z^n.$$

Corollary 2.3 If an analytic function $f \in A$ belonging to the class $M(\alpha,\beta,\mu,\lambda,\delta)$ then

$$|a_n| \leq \frac{(1-\delta)}{(n-\delta)\big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\big)^m}, \quad n \geq 2.$$

Similarly we proved Theorem 2.2.

3 Growth and Distortion Theorems

Theorem 3.1 If an analytic function f belonging to the class $M(\alpha,\beta,\mu,\lambda,\delta)$ then we have

$$|z|-\frac{(1-\delta)}{(2-\delta)}(\frac{\alpha+\mu+\lambda+\beta}{\alpha+\beta})^m|z|^2\le|f(z)|\le|z|+\frac{(1-\delta)}{(2-\delta)}(\frac{\alpha+\mu+\lambda+\beta}{\alpha+\beta})^m|z|^2.$$

Theorem 3.2 If an analytic function f belonging to the class $N(\alpha,\beta,\mu,\lambda,\delta)$ then we have

$$|z|-\frac{(1-\delta)}{2(2-\delta)}(\frac{\alpha+\mu+\lambda+\beta}{\alpha+\beta})^m|z|^2\le|f(z)|\le|z|+\frac{(1-\delta)}{2(2-\delta)}(\frac{\alpha+\mu+\lambda+\beta}{\alpha+\beta})^m|z|^2.$$

Theorem 3.3 If an analytic function f belonging to the class $M(\alpha,\beta,\mu,\lambda,\delta)$ then we have

$$1-\frac{2(1-\delta)}{(2-\delta)}(\frac{\alpha+\mu+\lambda+\beta}{\alpha+\beta})^m|z|\le|f'(z)|\le 1+\frac{2(1-\delta)}{(2-\delta)}(\frac{\alpha+\mu+\lambda+\beta}{\alpha+\beta})^m|z|.$$

Theorem 3.4 If an analytic function f belonging to the class $N(\alpha,\beta,\mu,\lambda,\delta)$ then we have

$$1-\frac{(1-\delta)}{(2-\delta)}(\frac{\alpha+\mu+\lambda+\beta}{\alpha+\beta})^m|z|\le|f'(z)|\le 1+\frac{(1-\delta)}{2(2-\delta)}(\frac{\alpha+\mu+\lambda+\beta}{\alpha+\beta})^m|z|.$$

Theorem 3.5 If an analytic function f belonging to the class $M(\alpha,\beta,\mu,\lambda,\delta)$ then we have

$$|z|-\frac{(1-\delta)}{(2-\delta)}|z|^2\le|f(z)|\le|z|+\frac{(1-\delta)}{(2-\delta)}|z|^2.$$

Theorem 3.6 If an analytic function f belonging to the class $N(\alpha,\beta,\mu,\lambda,\delta)$ then we have

$$|z|-\frac{(1-\delta)}{2(2-\delta)}|z|^2\le|f(z)|\le|z|+\frac{(1-\delta)}{2(2-\delta)}|z|^2.$$

Proof Let $f\in M(\alpha,\beta,\mu,\lambda,\delta)$ then

$$\sum_{n=2}^{\infty}(n-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m|a_n|\le 1-\delta,$$

therefore

$$(2-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)+\beta}\Big)^m\sum_{n=2}^{\infty}|a_n|\le\sum_{n=2}^{\infty}(n-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m|a_n|\le 1-\delta,$$

implies

$$(2-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)+\beta}\Big)^m\sum_{n=2}^{\infty}|a_n|\le 1-\delta.$$

Since

$$|f(z)| \leq |z| + \sum_{n=2}^{\infty} |a_n||z|^2,$$

hence we get

$$|f(z)| \quad \leq \quad |z| + \frac{(1-\delta)}{(2-\delta)}(\frac{\alpha+\mu+\lambda+\beta}{\alpha+\beta})^m |z|^2.$$

Similarly we proved that

$$|f(z)| \quad \geq \quad |z| - \frac{(1-\delta)}{(2-\delta)}(\frac{\alpha+\mu+\lambda+\beta}{\alpha+\beta})^m |z|^2.$$

Similarly we proved the remaining the Theorems.

4 Extreme Points

Theorem 4.1 If $f_1(z) = z$ and $f_i(z) = z + \frac{(1-\delta)}{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m} z^i$ where $i = 2, 3, 4, \cdots$. Then $f \in M(\alpha, \beta, \mu, \lambda, \delta)$ if and only if it can be expressed in the form $f(z) = \sum_{i=1}^{\infty} \lambda_i f_i(z)$ where $\lambda_i \geq 0$ and $\sum_{i=1}^{\infty} \lambda_i = 1$.

Theorem 4.2 If $f_1(z) = z$ and $f_i(z) = z + \frac{(1-\delta)}{n(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m} z^i$ where $i = 2, 3, 4, \cdots$. Then $f \in N(\alpha, \beta, \mu, \lambda, \delta)$ if and only if it can be expressed in the form $f(z) = \sum_{i=1}^{\infty} \lambda_i f_i(z)$ where $\lambda_i \geq 0$ and $\sum_{i=1}^{\infty} \lambda_i = 1$.

Proof Let

$$f(z) = \sum_{i=1}^{\infty} \lambda_i f_i(z)$$

implies

$$f(z) = \lambda_1 z + \sum_{i=2}^{\infty} \lambda_i f_i(z)$$

or

$$f(z) = \lambda_1 z + \sum_{i=2}^{\infty} \lambda_i (z + \frac{(1-\delta)}{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m} z^i),$$

$$f(z) = \lambda_1 z + \sum_{i=2}^{\infty} \lambda_i (z) + \sum_{i=2}^{\infty} \lambda_i (\frac{(1-\delta)}{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m} z^i)$$

$$f(z) = \sum_{i=1}^{\infty} \lambda_i (z) + \sum_{i=2}^{\infty} \lambda_i (\frac{(1-\delta)}{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m} z^i)$$

or

$$f(z) = z + \sum_{i=2}^{\infty} \lambda_i \left(\frac{(1-\delta)}{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m} z^i \right).$$

By using Theorem 2.1 we get

$$\sum_{i=2}^{\infty} \lambda_i \left(\frac{(1-\delta)}{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m} (n-\delta) (\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta} \right)^m = \sum_{i=2}^{\infty} \lambda_i (1-\delta) < (1-\delta)$$

implies $f \in M(\alpha, \beta, \mu, \lambda, \delta)$. Conversely we suppose that $f \in M(\alpha, \beta, \mu, \lambda, \delta)$ then by using Corollary 2.3 we have

$$|a_n| \leq \frac{(1-\delta)}{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m}, \quad n \geq 2,$$

we consider

$$\lambda_i = \frac{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m}{(1-\delta)} a_i, \; n \geq 2$$

then $f(z) = \sum_{i=1}^{\infty} \lambda_i f_i(z)$. Hence proved. Similarly we proved the second theorem.

5 Integral Means Inequalities

For any two functions f and g analytic in $\mathbb{U}$, f is said to be subordinate to g in $\mathbb{U}$ denoted by $f \prec g$ if there exists an analytic function w defined $\mathbb{U}$ satisfying $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = g(w(z))$, $z \in \mathbb{U}$.

In particular, if the function g is univalent in $\mathbb{U}$, the above subordination is equivalent to $f(0) = g(0)$ and $f(U) \subset g(U)$. In 1925, Littlewood proved the following Subordination Theorem.

Theorem 5.1 If f and g are any two functions, analytic in $\mathbb{U}$ with $f \prec g$ then for $\mu > 0$ and $z = re^{i\theta}$ $(0 < r < 1)$

$$\int_0^{2\pi} |f(z)|^{\mu} d\theta \leq \int_0^{2\pi} |g(z)|^{\mu} d\theta.$$

Theorem 5.2 Let $f \in M(\alpha, \beta, \mu, \lambda, \delta)$ and f_k be defined by

$$f_n(z) = z + \frac{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m}{(1-\delta)} z^n, \; n \geq 2.$$

If there exists an analytic function $w(z)$ given by

$$[w(z)]^{n-1} = \frac{(1-\delta)}{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m} \sum_{n=2}^{\infty} a_n z^{n-1}$$

then for $z = re^{i\theta}$ and $0 < r < 1$

$$\int_0^{2\pi} |f(re^{i\theta})|^\mu d\theta \leq \int_0^{2\pi} |f_n(re^{i\theta})|^\mu d\theta.$$

Theorem 5.3 Let $f \in N(\alpha, \beta, \mu, \lambda, \delta)$ and f_k be defined by

$$f_n(z) = z + \frac{n(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m}{(1-\delta)} z^n,\ n \geq 2.$$

If there exists an analytic function $w(z)$ given by

$$[w(z)]^{n-1} = \frac{(1-\delta)}{n(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m} \sum_{n=2}^{\infty} a_n z^{n-1}$$

then for $z = re^{i\theta}$ and $0 < r < 1$

$$\int_0^{2\pi} |f(re^{i\theta})|^\mu d\theta \leq \int_0^{2\pi} |f_n(re^{i\theta})|^\mu d\theta.$$

Proof Since

$$\int_0^{2\pi} |f(re^{i\theta})|^\mu d\theta \leq \int_0^{2\pi} |f_n(re^{i\theta})|^\mu d\theta,$$

implies

$$\int_0^{2\pi} |z + \sum_{n=2}^{\infty} a_n z^n|^\mu d\theta \leq \int_0^{2\pi} |z + \frac{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m}{(1-\delta)} z^n|^\mu d\theta,$$

it is true only if

$$1 + \sum_{n=2}^{\infty} a_n z^{n-1} \prec 1 + \frac{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m}{(1-\delta)} z^{n-1}.$$

We consider

$$1 + \sum_{n=2}^{\infty} a_n z^{n-1} = 1 + \frac{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m}{(1-\delta)} [w(z)]^{n-1},$$

implies

$$[w(z)]^{n-1} = \frac{(1-\delta)}{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m} \sum_{n=2}^{\infty} a_n z^{n-1}$$

implies $w(0) = 0$ and

$$\left|[w(z)]^{n-1}\right| = \frac{(1-\delta)}{(n-\delta)(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta})^m} \sum_{n=2}^{\infty} |a_n||z^{n-1}|.$$

Hence by using Theorem 2.1 we have $|[w(z)]^{n-1}| < 1$ as required. Similarly we proved theorem 5.3.

6 Hadamard Product

Theorem 6.1 Let $f, g \in M(\alpha, \beta, \mu, \lambda, \delta)$ where $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$ then $f * g \in M(\alpha, \beta, \mu, \lambda, \delta)$.

Theorem 6.2 Let $f, g \in N(\alpha, \beta, \mu, \lambda, \delta)$ where $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$ then $f * g \in N(\alpha, \beta, \mu, \lambda, \delta)$.

Proof Since $f, g \in M(\alpha, \beta, \mu, \lambda, \delta)$ therefore by using Theorem 2.1 we have

$$\sum_{n=2}^{\infty}(n-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m |a_n| \leq 1-\delta,$$

and

$$\sum_{n=2}^{\infty}(n-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m |b_n| \leq 1-\delta.$$

Because we know that

$$\sum_{n=2}^{\infty}(n-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m |b_n||a_n| \leq \sum_{n=2}^{\infty}(n-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m |a_n| \leq 1-\delta,$$

implies

$$\sum_{n=2}^{\infty}(n-\delta)\Big(\frac{\alpha+\beta}{\alpha+(\mu+\lambda)(n-1)+\beta}\Big)^m |b_n||a_n| \leq 1-\delta,$$

again by using Theorem 2.1 we conclude that $f * g \in N(\alpha, \beta, \mu, \lambda, \delta)$. Similarly we proved the second theorem .

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