Underlying System Using Dynamic Clustering and Structural Similarity

Hernando Castañeda M, Eliezer Colina M, Wladimir Rodríguez, Carlos Parra O

Abstract— **we propose a new perspective on the identification of linear dynamic system using structural similarity. The proposal consists in the meaningful exploration of each model, specifically behavior of the state variable.**

The decomposition of the behavior of a state variable in different modes of behavior of a system, each one has a different set of weights and shows different patterns of behavior. These weights are more significant than eigenvalue to develop a new technique for identifying linear system and invariants over time.

We use two methods based on different areas of knowledge such as linear algebra and statistics. This paper is a conceptual proof that enriches the implementation and validity not only from point of view algorithmic likewise physic mathematical.

I. Introduction

THIS document , we show how the behavior of any state variable in a linear system can be broken down into different modes of behavior, each being characterized by an eigenvalue. This paper is concerned with linear system, but it is hoped that it will enriched with new techniques for non linear systems. The temporal trajectory of a state variable i

$$x_i(t) = w_{i1}m_1(t) + ... + w_{ij}m_j(t) + ... + w_{in}m_n(t) + u_i \qquad (1)$$

Where $x_i(t)$ is the value of a state variable i in the instant t; w_{ij} is a constant term which represents the significance mode j to the variable i; m_{ij} is the value of j^{th} mode behavior in time t; u_i is a constant term. The mode of behavior of a linear system is a function of the eigenvalue of the Jacobian matrix which characterized the system (Oagata 1990).

If the eigenvalue do not have an imaginary part, the part of the behavior mode is expressed by the first answer of the last equation:

$$m_j = \begin{cases} \exp\ \mathrm{Re}\left[\lambda_j\right] t \quad \text{si } \mathrm{Im}\left[\lambda_j\right] = 0 \\ \exp\ \mathrm{Re}\left[\lambda_j\right] t \ \ sen\ \mathrm{Im}\left[\lambda_j\right] t + \theta \ = 0 \text{ case other} \end{cases} \qquad (2)$$

If the eigenvalues do not have an imaginary part, the part of the behavior mode is expressed by the first answer of the last equation (2) and is characterized by growth exponential function if the real part of the eigenvalue is positive y decrease exponential function if the real part of the eigenvalue is negative.

If an eigenvalue has an imaginary part that is different from zero, this means that the two eigenvalues are a conjugated pair (with the same real part) and together they generate the oscillating mode represented by the second expression of the last equation 13 .

$$x_i(t) = w_{i1}m_1(t) + ... + w_{ij}m_j(t) + ... + w_{in}m_n(t) + u_i$$

If the real part of the conjugated pair of eigenvalue is positive, an expanded oscillation mode is produced. If it is equal to zero a sustained oscillation mode is produced and if it is negative a dampened oscillation mode is produced. (See figure 1)

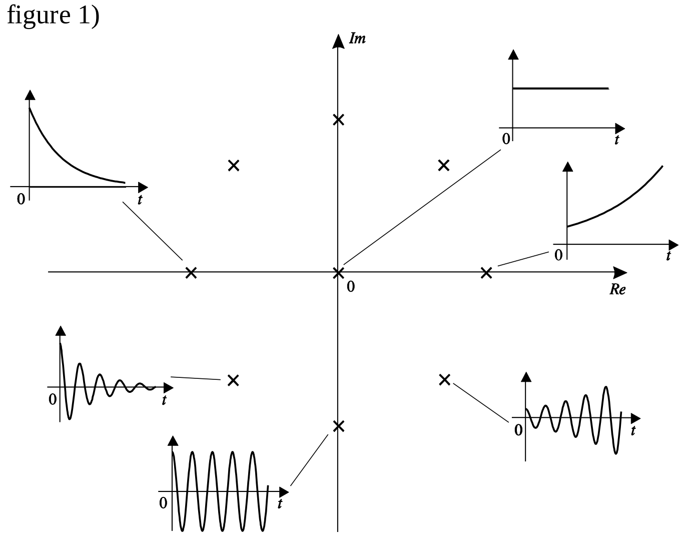

Fig 1 Eigenvalue Placement in Complex plane

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The breakdown of the temporary trajectory of state variable info behavior modes produces a useful set of diagnostics, not only to understand the sources of behavior of the variables, but also to identify the degree of interaction between system variables. Furthermore, the significance of the behavior mode of variable w_{ij} can also be used as a way of identifying the elements the structure responsible for the observed behavior 12 .

The aforesaid is articulated through the sensitivity valuation between the weight model and gain link model. The gain link between two variables is defined as the derived partial of the output variables with regards to the input variables $g_a = \partial a / \partial b$ and also the elasticity of the weight for a gain is defined as the relationship between a fractional change in the weight and a fractional change in the gain, for example $\varepsilon = \dfrac{\frac{\partial w}{w}}{\frac{\partial g}{g}}$.

II. Analysis of the Eigenvalues of Lineal Models

We can breakdown the temporal trajectories of a state variable info many behavior modes. The temporal trajectories of a state variable are a mathematical function that specifies the value of the state variables in any instant of time. The departure point is the structure of the model, which in the case of linear models can be represented by the following compact matrix equation:

$$\dot{\mathbf{x}}(t) = \mathbf{G}\mathbf{x}(t) + \mathbf{b} \qquad (3)$$

Where $\mathbf{x}$ the vector of the state variables is, $\dot{\mathbf{x}}$ is the vector of the first derivatives of the state variable (rates). $\mathbf{b}$ Is a constant vector and $\mathbf{G}$ is the Jacobian matrix or gain matrix specified this $\mathbf{G}_{\mathrm{ij}} = \dfrac{\partial \dot{x}_i}{\partial x_j}$. In linear system $\mathbf{G}$ is constant at least when the system is not lineal, where it is a function of the state variables and external entrance and consequently varies in time. $\mathbf{G}$ Is a constant in linear systems with zero variables or external constant, with the exception of non linear systems 12

If we differentiating equation (4) with respect to time we find the expression follow:

$$\ddot{\mathbf{x}}(t) = \mathbf{G}\dot{\mathbf{x}}(t) \qquad (4)$$

Where $\ddot{\mathbf{x}}$ is the vector curvature (the vector of the second derivates of the state variables). The gain matrix $\mathbf{G}$ relates the slope vector with the curvature vector in a standard space n- dimension in real.

The solution for a system of differential equations, specified by equation (4) gives the temporal trajectories of the system slope vector. The eigenvalue model will be use to resolve slope differential equations (Luenberg, 1979) for the temporary trajectory.

The n Eigen values and their right eigenvectors associated to the gain matrix $\mathbf{G}$ are defined as $\mathbf{G}\mathbf{r}_k = \lambda_k \mathbf{r}_k$.

The case of absence means having different eigenvalue and thus the right eigenvectors are linearly independent (Luenberg 1979) and cross in the space of n-dimension $\mathbb{R}^n$. Consequently, the slope vector can be expressed as a linear combination of right eigenvectors as :

$$\dot{\mathbf{x}}(t) = \alpha_1(t)\mathbf{r}_1 + \alpha_2(t)\mathbf{r}_2 + \ldots + \alpha_n(t)\mathbf{r}_n \qquad (4)$$

Where α_k the slope vector components are in the new system of coordinates and $\mathbf{r}_i$ are the constant sets of eigenvectors. The differential equation solution of (5) with respect to time produces the components $\dot{\alpha}_k$ the curvature vectors in the new system of coordinates as:

$$\ddot{\mathbf{x}}(t) = \alpha_1(t)\lambda_1\mathbf{r}_1 + \alpha_2(t)\lambda_2\mathbf{r}_2 + \ldots + \alpha_n(t)\lambda_n\mathbf{r}_n \qquad (5)$$

It is clear that only the determining factor of dynamic toward a particular coordinates, for example an eigenvector is the eigenvalue associated with the same coordinate. By substituting the solution for dynamic behavior of each α_k in the equation (5) a temporal trajectory of slope vectors is produced toward the dimension of own space,

$$\dot{\mathbf{x}}(t) = \alpha_1^0(t)e^{\lambda_1(t-\tau)}\mathbf{r}_1 + \ldots + \alpha_n^0(t)e^{\lambda_n(t-\tau)}\mathbf{r}_n \qquad (6)$$

If we integrate the former equation of slope trajectories with respect to time (from τ time to t time) and defining $\mathbf{w_k} = (\alpha_k^0 / \lambda_k)\mathbf{r}_k$ the following expression is obtained:

$$\mathbf{x}(t) = \mathbf{w_1}e^{\lambda_1(t-\tau)} + \mathbf{w_2}e^{\lambda_2(t-\tau)} + \ldots + \mathbf{w_n}e^{\lambda_n(t-\tau)} + \mathbf{u}$$

This decomposes the state trajectories into many forms of behavior, which are characterized by an eigenvalue.

III. Parametric Structure of the Decomposition of Dynamic System Behavior

In the first part of this document, a mathematical equation was developed for the state trajectory behavior of dynamic system (see equation 7). In this section, the origin of each component of the state trajectory will be identified 12 .

In an interpretation of figure 2, the basic component is:

The eigenvalue $\boldsymbol{\lambda}_k$, the right eigenvector $\mathbf{r}_k$ the initial values of the slopes $\boldsymbol{\alpha}_k$ and parameters.

We notice the use of delay links to indicate eigenvalue, right eigenvectors and alphas controls of the future trajectories of the state variables

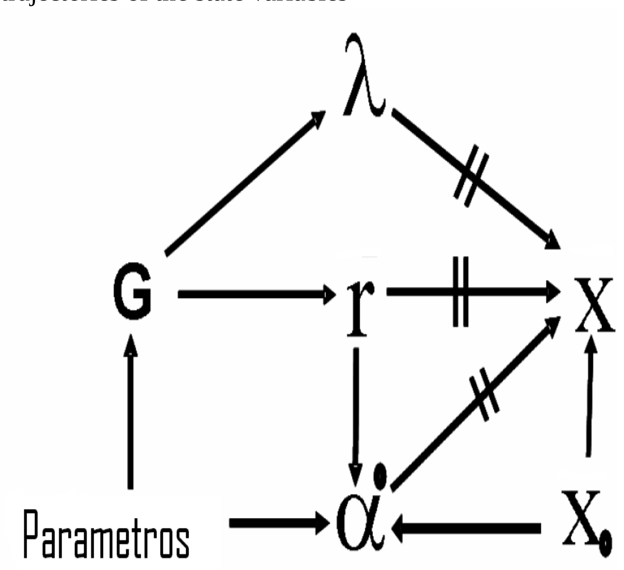

Fig 2 Parametric Structure of Identification

An analysis of the eigenvalue will be included as they can play an import role in modeling of behavior. For example, if was shown that a simple linear model with a positive feedback cycle can generate an exponential behavior decrease, rather than an exponential growth ; if the initial slope vector was orthogonal with the right eigenvector associated with el positive eigenvalue (Saleh and Davinsen ,2001).

When observing the eigenvalues, we see that they only originate from the model structure; or more specifically from the gain matrix $\mathbf{G}$. The gain matrix is used as a condensed representation of the structure of the model and was used in section I as a starting point for the state trajectory decomposition in different modes of behaviors.

Furthermore , it can be observed that in linear models , the gain matrix $\mathbf{G}$, depends on the parameters of the model (constant in the model) , although , for each eigenvalue it is possible to formulate the eigenvalues (depend variable) and the parameters of the model (independent variable) however for simplification instead of formulating a simple complicated function , relating an eigenvalue with all the parameters of the model, it is possible to develop many mathematical functions , where each function relates an eigenvalue to a single parameter 1 . In our investigation we used Matlab Toolbox to automate the aforementioned process. Where $\mathbf{x}$ the vector of the state variables is, $\dot{\mathbf{x}}$ is the vector of the first derivatives of the state variable (rates). $\mathbf{b}$ Is a constant vector and $\mathbf{G}$ is the Jacobian matrix or gain matrix specified this $\mathbf{G}_{ij} = \frac{\partial \dot{x}_i}{\partial x_j}$. In linear system $\mathbf{G}$ is constant at least when the system is not lineal, where it is a function of the state variables and external entrance and consequently varies in time. $\mathbf{G}$ Is a constant in linear systems with zero variables or external constant, with the exception of non linear systems 12

Similarly, the eigenvector only originates from the gain matrix. It is possible to formulate mathematical functions relating any eigenvector (dependent variable) for any parameter (independent variables).

The initial value of each alpha (for example α_k^0) represents the initial slope vector projection along the specific right eigenvector (for example a specific coordinate in own space). For example, the initial value of the alpha are depend in the initial value of net rates (values at the beginning of the simulation) and the right eigenvectors. To sum up, for each component in state trajectory (equation 7) it is possible to formulate mathematical functions relating the component with any parameter. This means that a compound mathematical function can be developed relating the future values in any given moment in time, of a state variable (dependent variable) with any parameters (independent variable). The partial differentiation in any moment in time of this compound function with respect to the parameter produces the future value sensitivity (of the state variable) with this parameter.

The state of transition of the dynamic system in the internal space and the mapping from the space of internal states to the space of observations is modeled by the following linear equations:

$$x_t = F^{(i)} x_{t-1} + g^{(i)} + w_t^{(i)}$$
$$y_t = H x_{t-1} + v_t$$

Where $F^{(i)}$ is a transition matrix; $g^{(i)}$ is a bias vector. H Is a transition matrix that defines the lineal projection from a space of internal state to the observation space, Notice that each dynamic system has, $F^{(i)}$ $g^{(i)}$ y $w_t^{(i)}$ individually. It is assumed that each $w^{(i)}$ is noise identifier and v has normal distribution $N_{x_t}\ 0, Q^{(i)}$ and $N_y\ 0, R$ respectively.

The classes of dynamic systems can be categorized by the eigenvalue of the transition matrix which determines answers of the input zero of the system. In other words, these eigenvalues determine the general behavior of patterns (trajectories) with temporary variation in the space of states.

A. Decomposition of Eigenvalue Starting From The Gain Matrix

The general class of dynamic pattern (corresponding to trajectories at point in the states space) of a linear dynamical system can be described by the eigenvalues of the gain matrix. For the concentration of the temporal evolution of states in dynamical system, it can be assumed that the bias and the noise process are zero in equation 1, using the decomposition of the eigenvalue of the gain matrix; we arrive at the following equations 6

$$G = E\,\Lambda\,E^{-1} = e_1,\ldots,e_n\ diag\ \lambda_1,\ldots,\lambda_n\ \ e_1,\ldots,e_n\ ^{-1} \tag{7}$$

The state in time t can be resolved with initial conditions x_0 as follows:

$$\begin{aligned} x_t &= F^t x_0 = \ E\,\Lambda\,E^{-1}\ ^T x_0 \\ E\,\Lambda^T\,E^{-1} x_0 &= \sum_{p=1}^{n} \alpha_p\ e_p\ \lambda_p^t \end{aligned} \tag{8}$$

Where λ_p and e_p are the corresponding eigenvalue and eigenvector. The weight value α_p is determined from the initial state x_0 by the determination on the complex plane as:

$$\lambda_1,\ldots,\lambda_n\ ^T = E^{-1} x_0 \tag{9}$$

From here, the general patterns of a system can be categorized by using the position of the Eigen values (poles) $\lambda_1,\ldots,\lambda_n$ on the complex plane. The determination of the oscillatory state is determined by its argument values (angles), according to the following rules:

Oscillation if at least one eigenvalue is negative or complex

No oscillating if all the eigenvalue are real numbers.

The absolute value of the eigenvalue determines the state of convergence or divergence of the form:

Diverge if at last one the eigenvalue is above one.

Converge if all the absolute values of the eigenvalue are less than one.

The system can generate pattern due to temporal variation if and only if $\left|\lambda_p\right| < 1\ for\ 1 \le p \le n$; this pattern converges at zero. (In control terms it is said that the system is stable)

The system can generate monotonic or cyclical patterns if the imaginary part of the eigenvalue is different to zero.

A. Identification Through The Estimation of The Gain Matrix

The identification of the system with no restrictions is conditioned a ranges temporal b, e are represented by the dynamic linear system D_i, thus the transition matrix can be estimated $F^{(i)}$ and the bias vector $g^{(i)}$ of the sequence $x_b^{(i)},\ldots,x_e^{(i)}$ of internal states. This problem of parameter estimation becomes a problem of minimization of error prediction 6.

This error prediction vector can be determined using the discrete equations for dynamical linear system and after having estimated the matrix of $F^{(i)}$ the bias vector $g^{(i)}$. The formulation is as follows:

$$\varepsilon_t = x_t^{(i)} - \ F^{(i)} x_{t-1}^{(i)} + g^{(i)} \tag{10}$$

Thus, the sum of the norms of the squares of all of the error vectors in the range b, e becomes:

$$\sum_{t=b+1}^{e} \left\| e_t \right\|^2 = \sum_{t=b+1}^{e} \left\| x_t^2 - \ F^{(i)} x_{t-1}^{(i)} + g^{(i)}\ \right\|^2 \tag{11}$$

Finally, the optimum values of $F^{(i)}$ and $g^{(i)}$ can be estimated by solving the following problem of least minimums square as:

$$F^{(*i)}, g^{(*i)} = \arg\ \min \min_{F^{(i)}, g^{(i)}} \sum_{t=b+1}^{e} \left\| e_t \right\|^2 \tag{12}$$

IV. Definition of the Problem

The principal motivation of this work is to develop some methods or techniques which allow the study of complex systems, in the sense of finding their underling structure or structural similarity with known systems.

This enables us to look for data structures and their classification into categories in such way that the similarity between structures of the same category is high and the different categories of similarity values low. Traditional approaches to system analysis –e.g. trying to find a mathematical model that describes output as a function of state variable and due to the fact that input perform poorly when dealing with complex systems.

This may be due to their nonlinear, time-varying nature or to uncertainty in the available measurement [1]. We can approach the analysis of dynamic systems in two different ways: the first is based on the existence of a state measuring mechanism in the form of a mathematical model; In the absence of such a measuring mechanism, we must resort to some perceptual mechanism, that allows us to perceive the underling structure of the system, based on the behavior of the dynamic system.

The similarity measure is one the possible perceptual mechanism that can be used to analyze such systems. One of the motivations of this dissertation is to discover ways to use structural similarity as mechanics to study dynamic systems.

The classic methods of recognition of patterns should be tuned to consider desirable problems from the dynamic point of view, that is to say the process of objects are described with sequences of temporary observations.

In the design of dynamic systems and analysis in the domain of time, the concept of states of a system is used; a dynamic system is usually modeled by a system of differential equations.

To obtain dynamic systems by differential equations that represent the relationship between the input variables $u_1(t), u_2(t), ..., u_p(t)$ and the output variables $y_0(t), y_1(t), ..., y_q(t)$, the intermediate variables receive the name of state variables $x_1(t), y_2(t), ..., x_n(t)$. A set of state variables in any instant determines the state of the system at this time [13].

If the current state of a system and the value of the variables are given for $t > t_0$, the behavior of the system can be described clearly. The state of the systems is a set of real numbers in such a way that the knowledge of these numbers and the values of the input variables provide the future state

$$
\begin{aligned}
G^{(i)}* &= \widehat{X}_1^{(i)} \widehat{X}_0^{(i)+} \\
u^{(i)} &= m_1 - G^{(i)*} m_1
\end{aligned}
$$

of the system and the values of the output variables by the equations that describe the dynamics of the system. The state variables determine the future behavior of the system when the current state of the system and the values of the input variables are known. The multidimensional space of observation induced by the state variables receives the name of space of states.

The solution of a system of differential equations can be represented by a vector $\mathbf{x}(t)$ that corresponds to a point in the state space in an instant of time t. This point moves in the space of states like steps of time. The appearance or the way to this point in the space of states is known like as trajectory of the system.

For an initial state and end state given an infinite number of input vectors exist that correspond to trajectories with start and end points. On the other hand, through a point on the state space only one trajectory passes.

Considering dynamic systems in the control theory, a lot of attention has been paid to adaptive control [1]. The main reason to introduce this area of investigation is to obtain controllers whose parameters can adapt to the changes in the dynamic process dynamic to perturbation characteristic.

V. RESULTS

There is an identification method know as identification of dynamic system with no restriction in the eigenvalue which allows us to estimate the gain matrix starting from an interval $[b, e]$ that represents a behavior mode of the state trajectory , specified by $x^{(i)}$. Taking the discrete equation form for dynamic system $x_i = Gx_{i+1} + u$, we begin our method with the following expressions:

$$
\begin{aligned}
\widehat{X}_0^{(i)} &= \left[x_b^{(i)} - m_0^{(i)}, ..., x_{e-1}^{(i)} - m_0^{(i)} \right] \\
\widehat{X}_1^{(i)} &= \left[x_{b+1}^{(i)} - m_1^{(i)}, ..., x_e^{(i)} - m_1^{(i)} \right]
\end{aligned}
\tag{13}
$$

Where $m_0^{(i)}$ and $m_1^{(i)}$ are the middle value of columns in $X_0^{(i)}$ and $X_1^{(i)}$ respectively and which are formulated in this way:

$$
\begin{aligned}
m_0^{(i)} &= \frac{1}{l-1} \sum_{t=b}^{e-1} x_t^{(i)} \\
m_1^{(i)} &= \frac{1}{l-1} \sum_{t=b+1}^{e} x_t^{(i)}
\end{aligned}
\tag{14}
$$

The gain matrix and the bias term can be calculated, for each interval in the state trajectory in the follow form:

$$
\begin{aligned}
G^{(i)}* &= \widehat{X}_1^{(i)} \widehat{X}_0^{(i)+} \\
u^{(i)} &= m_1 - G^{(i)*} m_1
\end{aligned}
$$

Where $X_0^{(i)+}$ a Moore Penrose [2] is generalized inverse (Moore Penrose pseudo inverse) of $X_0^{(i)}$ the inverse matrix X^+ can be defined as:

$$
\begin{aligned}
X^+ &= \lim_{\delta^2 \to 0} X^T \left(XX^T + \delta^2 I \right)^{-1} \\
&= \lim_{\delta^2 \to 0} \left(XX^T + \delta^2 I \right)^{-1} X^T
\end{aligned}
\tag{15}
$$

Where I is the unit matrix and δ is real value different to zero [6].

In summary, one can say that the method appears to be very useful for both accurate identification of distinct behavior modes and for quickly identifying coherent pieces of structures involved a structural similarity.

The behavior of fermented process can be divided in phases, which are also indicated in figures 6, 7: a period of exponential growth, balancing growth, exponential decline and balancing decline.

The first example describes identification of the highly nonlinear pressure dynamic in a laboratory of fermentation [13]. The fermentation process under consideration consists of a 40 l tank containing 25 l of water. At the bottom of tank, air is fed into the water at a specified constant flow rate. The air pressure above the water level $y(t)$ is controlled by a one valve $u(t)$. Nonlinearities are both due to the valve characteristic y the air compression curve. The two data sequence used for identification is shown in Figure 3.

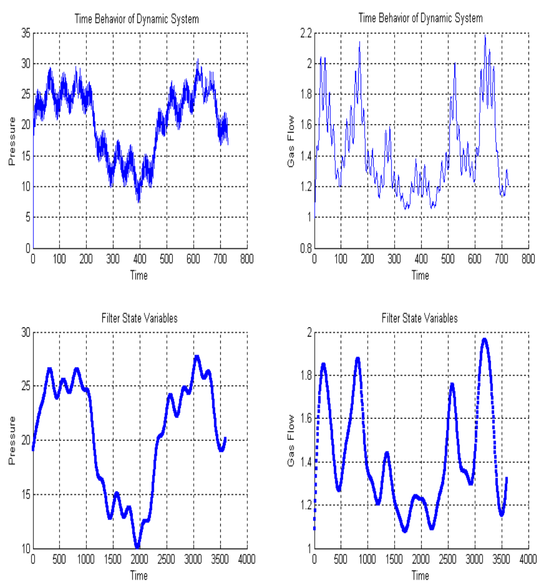

Fig 3 State Variables and Smooth Variables

The number of clusters and the location of cluster center at moment in time constitute the cluster structure. If in the course of time the number clusters and the locations of clusters centers vary, then one has to deal with the dynamic cluster structure.

Its temporal development is represented by trajectories of the cluster centers in the feature space (Figure 5). Changes in the cluster structure correspond to change of a state, or behavior, of a system under study and will also be referred to a structural change.

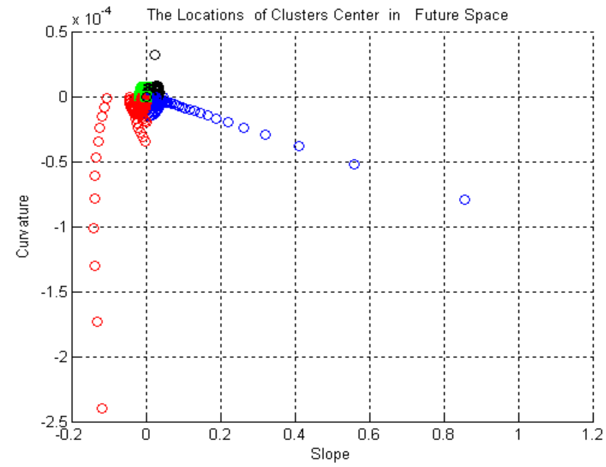

Fig 4 Changes in the Cluster Structure

Consider this dynamic complex system that can assume different state in the course of time. Each state of the system at moment time represents an object for classifications .as state, a dynamic system is describe by curvature and slope variables characterizing its dynamic behavioral. Each object dynamic is a temporal sequence of observations and described by discrete function of time is called a trajectory on an object. Thus , based in the form of trajectories was chosen as a criterion of similarity between trajectories , then five clusters of dynamic objects can be distinguished (Figure 5) the labels are : red, green, blue cyan , magenta.

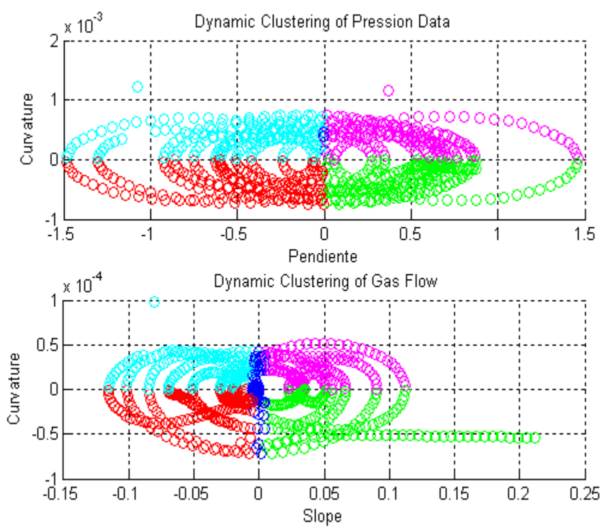

Fig 5 Dynamic Clustering in Feature Space

In the following example the structural features of the system of fermentation will be examined which may have critical behaviors.

Comparing system dynamic during different periods of operation, (Figure 6 and 7) different cluster of system can be distinguished: for example, in the first instant of time, instant of time, in both trajectories is assigned to a red green; in an instant of time later, in the first trajectories is assigned to a cyan cluster and in the second trajectory is assigned to the red cluster. Hence, the dynamic of this situation shows itself in changing cluster structure and in the transition of dynamic object represented by trajectories between clusters.

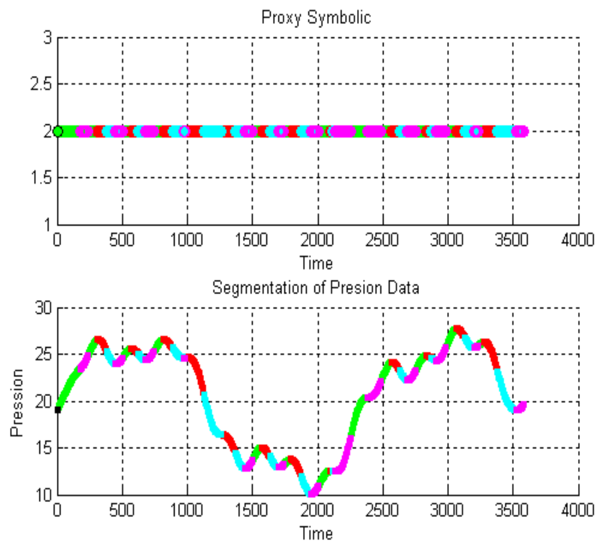

Fig 6 Temporal Behavior of Pressure Clusters

The purpose of decomposing of a fermentation system is satisfied by the dynamic clustering results as shows in the following figure

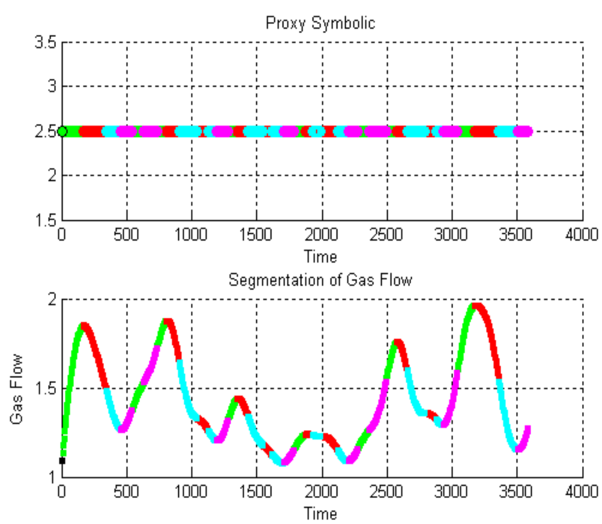

Fig 7 Dynamic Clustering in Gas Flow System

This dynamic clustering reveals a decomposition of the extended dynamic oscillatory system in increasing and decreasing asymptotic behavior corresponding to first with negative feedback.

The previous statement whilst being true serves as a basis to know the system structure through a lineal combination of behavior modes specified by the values of the real and complex eigenvalue, as show in the following graphs:

Using eigenvalue analysis, however, it is possible to characterize the behavior with more precision. Figure 8, 9 shows modes specified by the values of the real and imaginary part of the system eigenvalue over time.

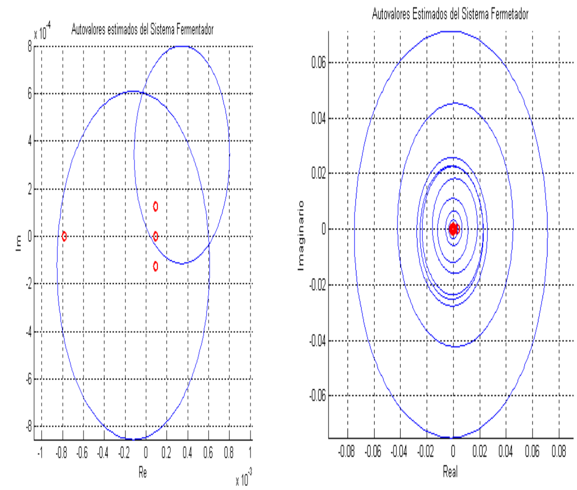

Fig 8 Eigenvalue in the System of Fermentation

Now, we will illustrate the behavior of the other segments which allows their mode of behavior to be established based on whether their eigenvalue are real or imaginary.

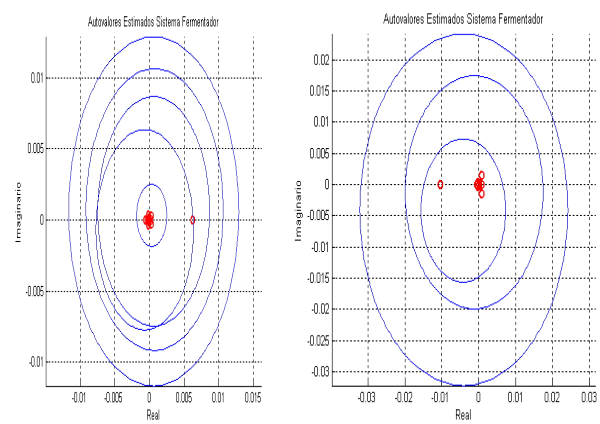

Fig 9 Eigenvalue in the Complex Plane

In these last result of the identification method using linear algebra an increasing exponential behavior is observed, similar to that of a first order system with positive feedback and an oscillatory behavior characterized by its two eigenvalue with complex value.

Identification results using statistics methods like principal component analysis (PCA) 11 will inform us about the behavior of system, its component and the R distribution of its eigenvalues for the fermented system. In figure 10 this result can see.

techniques which are more or less effective are know in the academic foundations of systems identification.

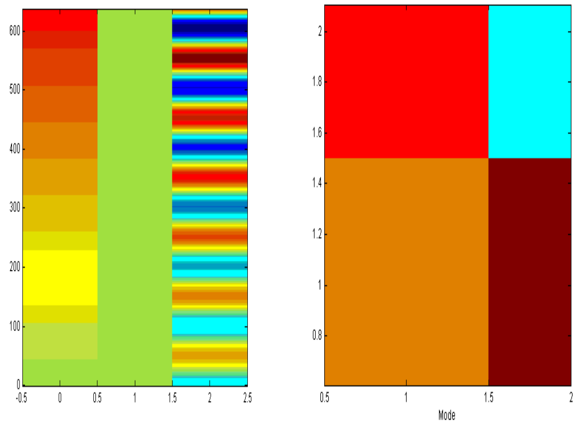

Fig 10 Grid of Eigenvalue for Principal Segment the Fermentation System

The PCA the light colors represent occurrences of high negative values and the dark colors represent occurrences of high positive values 11 . Thus, areas with a similar tone mean that their coefficient has similar values.

VI. Conclusion

The previous results need to part from explicit data taken from real world abstraction methods and transformed into structural features through geometric descriptors for example structural similarity.

The features space considered is a space of slope values vs. curvature values as established in equation 3. Once the feature vector is specified, which is no more than the curvature and slope vector from trajectories of states variable of dynamic system. These values are inputs for the process of dynamic clustering Castañeda Colina (DCCC) which partition the primary state variable trajectories into their respective behaviors. This allows get the segment to be mapped in temporal space for the identification process no restrict.

All of the values of state variables need to be considered in this features space to estimate the gain matrix, Moore Penrose method is used accompanied by a regularization coefficient, the matrix factors being curvature vector and slopes vector of the state variables.

Once the matrix has been estimated , if is the input in the process known as Gershgorin´s circle methods , which enables the position of the eigenvalue to be found ,it process becomes a clustering process of the eigenvalue.

Furthermore, by using the statistic method know as principal component analysis the behavior of the eigenvalue can be illustrated for each mode of behavior.

We consider that our model is transparent and a detailed analysis will prove this, although we know that numerous techniques which are more or less effective are know in the academic foundations of systems identification.

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