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The Bishop Darboux Rotation axis of the Spacelike Curves with a Spacelike Principal Normal in Minkowski 3-Space

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Abstract

The Bishop darboux rotation for Spacelike curves with a spacelike Principal Normal in Minkowski 3-space E_1^3 is decomposed into two simultaneous rotations. The axes of these simultaneous rotations are joined by a simple mechanism. One of these axes is a parallel of the tangent vector of the timelike curve, the direction of the other is the direction of the Bishop darboux vector of the Spacelike curves with a spacelike principal normal.

key words. Bishop Frame, Parallel transport frame, Bishop darboux vector, Bishop darboux rotation axis; Spacelike curves, Minkowski 3-Space

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1 Preliminaries

Let $IR^3 = \{(x_1, x_2, x_3) : x_1, x_2, x_3 \in IR\}$ be a 3-dimensional vector space, and let $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ be two vectors in IR^3. The Lorentz scalar product of x and y is defined by

$$\langle x, y\rangle_L = x_1y_1 + x_2y_2 - x_3y_3,$$

$IE_1^3 = \left(R^3, \langle x, y\rangle_L\right)$ is called 3-dimensional Lorentzian space, Minkowski 3-Space or 3- dimensional Semi-Euclidean space. The vector x in IE_1^3 is called a spacelike vector, null vector or a timelike vector if $\langle x, x\rangle_L > 0$ or $x = 0$, $\langle x, x\rangle_L = 0$ or $\langle x, x\rangle_L < 0$, respectively. For $x \in IE_1^3$, the norm of the vector x defined by $\|x\|_L = \sqrt{|\langle x, x\rangle_L|}$, and x is called a unit vector if $\|x\|_L = 1$. For any x, $y \in IE_1^3$, Lorentzian vectoral product of x and y is defined by

$$x \wedge_L y = (x_2y_3 - x_3y_2, x_1y_3 - x_3y_1, x_1y_2 - x_2y_1).$$

Denote by $\{T, N, B\}$ the moving Frenet frame along the curve α. Then T, N and B are the tangent, the principal normal and vector binormal of the curve α respectively. If α is a spacelike curve with a spacelike principal normal, then this set of orthogonal unit vectors, known as the Frenet-Serret frame, have properties

$$T^{'} = \kappa N, N^{'} = -\kappa T + \tau B, \ \ B^{'} = \tau N$$

$$\langle T, T\rangle_L, \langle N, N\rangle_L = 1, \ \langle B, B\rangle_L = -1$$

[1].

2 Introduction

The Bishop frame or parallel transport frame is an alternative approach to defining a moving frame that is well defined even when the spacelike curve with a spacelike principal normal has vanishing second derivative.we can parallel transport an orthonormal frame along a spacelike curve with a spacelike principal normal curve simply by parallel transporting each component of the frame. The parallel transport frame is based on the observation that, while $T(s)$ for a given spacelike curve with a spacelike principal normal curve model is unique, we may choose any convenient arbitrary basis $(N_1(s), N_2(s))$ for the remainder of the frame, so long as it is in the normal plane perpendicular to $T(s)$ at each point. If the derivatives of $(N_1(s), N_2(s))$depend only on $T(s)$ and not each other we can make $N_1(s)$ and $N_2(s)$ vary smoothly throughout the path regardless of the curvature. Therefore, we have the alternative frame equations

$$\begin{bmatrix} T' \\ N_1' \\ N_2' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & -k_2 \\ -k_1 & 0 & 0 \\ -k_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} T \\ N_1 \\ N_2 \end{bmatrix}. \tag{1.1}$$

where

$$\kappa(t) = \sqrt{\left|k_1^2 - k_2^2\right|},$$

$$\theta(t) = \arg\tanh\left(\frac{k_2}{k_1}\right),$$

$$\tau(t) = \pm\frac{d\theta(s)}{ds},$$

[5], so that k_1 and k_2 effectively correspond to a Cartesian coordinate system for the polar coordinates κ, θ with $\theta = \pm\int \tau(s)ds$. The orientation of the parallel transport frame includes the

arbitrary choice of integration constant θ_0, which disappears from τ (and hence from the Frenet frame) due to the differentiation.

Now, we will define Bishop darboux vector. The Vectors $T(s), N_1(s), N_2(s)$ change while a point on the spacelike curve with a spacelike principal normal drawing the spacelike curve with a spacelike principal normal. Hence these vectors constitute of spherical images of the spacelike curve with a spacelike principal normal. Assume that Bishop frame $\{T(s),\ N_1(s),\ N_2(s)\}$of the spacelike curve with a spacelike principal normal makes an instantaneous helix motion about an axis at each s time. This axis is called Bishop darboux axis corresponding s parameter at point $\alpha(s)$. The vector giving oriented and direction of this axis is called Bishop darboux vector at point $\alpha(s)$ of the spacelike curve with a spacelike principal normal.

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These equations form a rotation motion with Bishop darboux vector,

$$\varpi = k_2 N_1 + k_1 N_2$$

[5].Also momentum rotation vector is expressed as follows:

$$\begin{aligned} T' &= \varpi \wedge_L T \\ N_1' &= \varpi \wedge_L N_1 \\ N_2' &= \varpi \wedge_L N_2. \end{aligned}$$

Bishop darboux rotation of Bishop frame can be separated into two rotation motions. The vector N_1 rotates with a $k_1$angular speed round the vector N_2, that is

$$N_1' = (k_1 N_2) \wedge_L N_1 = k_1 (N_2 \wedge_L N_1) = -k_1 T$$

and the vector N_2 rotates with a k_2 angular speed round the vector N_1, that is

$$N_2' = (k_2 N_1) \wedge_l N_2 = k_2 (N_1 \wedge_L N_2) = k_2 T.$$

The separation of the rotation motion of the momentum.Bishop darboux axis into two rotation motions can be indicated as such: The vector $E = \frac{\varpi}{\|\varpi\|_L}$ rotates with

$$W = \frac{k_1' k_2 - k_1 k_2'}{\left|k_1^2 - k_2^2\right|}$$

a speed round the tangent vector T, also

$$E^{'}=\left(\frac{\varpi}{\left\|\varpi\right\|_L}\right)^{'}=(W.T)\wedge_L\frac{\varpi}{\left\|\varpi\right\|_L}$$

and the tangent vector T rotates with a $\|\varpi\|$ angular speed round $\frac{\varpi}{\|\varpi\|}$. Bishop darboux axis, also

$$T'=\varpi\wedge_L T.$$

From now on we shall do a further study of momentum Bishop darboux axis. We obtain the unit vector E

$$E=\frac{\varpi}{\left\|\varpi\right\|_L}=\frac{k_2N_1+k_1N_2}{\sqrt{\left|k_1^2-k_2^2\right|}}.$$

from Bishop darboux vector,

$$\varpi'=k_2'N_1+k_1'N_2.$$

Differentiation of E,

$$E'=\left(\frac{\varpi}{\left\|\varpi\right\|_L}\right)'=\frac{\varpi'\left\|\varpi\right\|_L-\varpi\left\|\varpi\right\|_L'}{\left(\left\|\varpi\right\|_L\right)^2}=\frac{(k_1'k_2-k_1k_2')}{\left|k_1^2-k_2^2\right|}\frac{(k_1N_1+k_2N_2)}{\sqrt{\left|k_1^2-k_2^2\right|}}$$

is found. From this,

$$E'=W\ (E\wedge_L T)+0.T+0.E$$

written. According to the Bishop frame,

$$T'=\left\|\varpi\right\|_L(E\wedge_L T)+0.T+0.E$$

and

$$\begin{aligned}(E\wedge_L T)' &= E'\wedge_L T+E\wedge_L T'\\ &= W\ \left[(E\wedge_L T)\wedge_L T\right]+\|\varpi\|\left[E\wedge_L(E\wedge_L T)\right]\\ &= W\ \left[-\langle T,E\rangle T+\langle T,T\rangle E\right]+\|\varpi\|\left[-\langle E,T\rangle E+\langle E,E\rangle T\right]\\ &= W.E+\|\varpi\|\langle E,E\rangle T\\ &= 0.(E\wedge_L T)+\varepsilon_1\|\varpi\|T+W.E\quad;\ \varepsilon_1=\pm1\end{aligned}$$

are obtained. These three equations are in the form of the Bishop frames that is

$$\begin{bmatrix} (E \wedge T)' \\ T' \\ E' \end{bmatrix} = \begin{bmatrix} 0 & \|\varpi\|_L & W \\ \varepsilon_1 \|\varpi\|_L & 0 & 0 \\ W & 0 & 0 \end{bmatrix} \begin{bmatrix} (E \wedge T) \\ T \\ E \end{bmatrix}$$

where the first coefficient $\|\varpi\|_L = \sqrt{\left|k_1^2 - k_2^2\right|}$ is > 0 and second coefficient

$$W = \frac{(k_1' k_2 - k_1 k_2')}{\left|k_1^2 - k_2^2\right|} = \frac{\left(\frac{k_1}{k_2}\right)'}{\left|1 - \left(\frac{k_1}{k_2}\right)^2\right|} \quad ; k_2 \neq 0 \ , \ |k_1| \neq |k_2|$$

related only to natural harmonic curvature $\frac{k_1}{k_2}$. Thus, the vectors $\{(E \wedge_L T), T, E\}$define a rotation motion together the rotation vector,

$$\varpi_1 = \varepsilon_1 \{-W.T + \|\varpi\|_L E\} = \varepsilon_1 (-W.T + \varpi) \text{ or } \pm (-W.T + \varpi)$$

Also momentum rotation vector is expressed as follows:

$$\begin{aligned} T' &= \varpi_1 \wedge_L T \\ (E \wedge_L T)' &= \varpi_1 \wedge (E \wedge_L T) \\ E' &= \varpi_1 \wedge_L E. \end{aligned}$$

Corollary 3.1 *This rotation motion of Bishop darboux axis can be separated into two rotation motions again. Here ϖ_1 rotation vector is the addition of the rotation vectors of the rotation motions. When continued in the similar way, the rotation motion of Bishop darboux axis is done in a consecutive manner. In this way the series of Bishop darboux vectors are obtained. That is*

$$\varpi_0 = \varpi, \varpi_1 ...$$

References

[1] Ahmet Yucesan, A. Ceylan Coken, Nihat Ayyildiz,"On the Darboux rotation axis of Lorentz space curve",Applied Mathematics and Computation 155 ,345-351, 2004

[2] Andrew J. Hanson, Hui Ma, "Parallel Transport Approach To Curve Framing", Indiana University, Techreports- TR425, January 11,1995

[3] Andrew J. Hanson, Hui Ma,“Quaternion Frame Approach to Streamline Visualization,” Ieee Transactions On Visualization And Computer Graphics, Vol. I , No. 2, June 1995

[4] Bishop, L.R., “There Is More Than One Way To Frame A Curve”, Amer. Math. Monthly, Volume 82,Issue 3, 246-251,1975

[5] B.Bukcu and M.K.Karacan,”Bishop Frame of The Spacelike Curve with a Spacelike Principal Normal in Minkowski 3-Space”,Commun.Fac.Sci.Univ.Ank.Series A1,Volume 57, Number 1, Pages:13-22, 2008

[6] B.Bukcu and M.K.Karacan,“The Bishop Darboux Rotation Axis of The Spacelike Curve in Minkowski 3-Space”, JFS, Volume 30, pp: 1-5, 2007

[7] O.Munaux, ”Cad Interface and Framework for Curve Optimisation Applications”,Cranfield University, PhD Thesis,September 2004

[8] W. Barthel, ”Zum Drehvorgang Der Darboux-Achse Einer Raumkurve”, J. Geometry 49,1994

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