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A CHARACTERIZATION OF CONTINUITY

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In [2] Kasahara introduced the concept of an operator associated to a topology. Following his approach we introduce two concepts that generalize those of almost-continuity and nearly-continuity of a function, and generalize the characterization of continuity given in [1].

Definition 1. ([2]) Let (X,τ) be a topological space and α an operator from τ to $P(x)$. We say that α in an operator associated to τ if

1. $\alpha(\emptyset) = \emptyset$
2. $U \subset \alpha(u)$ for every $u \in \tau$

Definition 2. Let (X,τ) and (Y,ρ) two topological spaces and α an operator associated to τ. Then a function $f:X \to Y$ is said to be almost α-continuous if for any open set V in Y, $f^{-1}(V) \subset \text{Int}\ \alpha(\text{Int}\ f^{-1}(V))$, and it is said that f is α-nearly continuous if for every open set in γ, $f^{-1}(V)$ is an open subset of $\alpha(\text{Int}\ f^{-1}(V))$ with the subspace topology.

Remark. If α is the closure operator, the above definitions coincide with that of almost continuity and nearly continuity respectively ([1]), and if α is the identity operator then each imply continuity.

Theorem. Let (X,τ) and (Y,ρ) be two topological spaces and α an operator associated to τ. A function $f:X\to Y$ is continuous if and only if it is almost α-continuous and α-nearly continuous.

Proof. Suppose f is continuous then for every open set V in Y the set $f^{-1}(V)$ is open. Then by (2) of definition 1, $f^{-1}(V) \subset \alpha(f^{-1}(V))$, then clearly $f^{-1}(V)$ $\text{Int}\,\alpha(\text{Int}\, f^{-1}(V))$. Also since $f^{-1}(V)$ is open in $\alpha(\text{Int}\, f^{-1}(V))$ then f is both almost α-continuous and α-nearly continuous.

Conversely, if V is an open set in Y then $f^{-1}(V)$ is a subset of $\text{Int}\,\alpha(\text{Int}\, f^{-1}(V))$ and an open subset of $\alpha(\text{Int}\,(f^{-1}(V)))$. Then there is an open subset U of X such that $f^{-1}(V) \subset U \subset \alpha(\text{Int}\,(f^{-1}(V))$ but that implies that $f^{-1}(V)$ is an open subset of U, which in fact implies that $f^{-1}(V)$ is an open subset of X therefore f is continuous.

References

[1] Jingcheng Tong, A characterization of continuity, Math Magazine, Vol. 65, N. 4(1992), 255-256.

[2] Kasahara, S., Operation-compact spaces, Math Japonica 24(1979), 97-105.